

# CONFINEMENT

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**Abstract**— In this paper i review briefly the confinement property of quarks in QCD.

**Index Terms**— Baryon, Confinement, Diquark, Gluon, Hadron, Quark, QCD.

## 1 INTRODUCTION

How can one be so confident of quark model when no one has ever seen an isolated one. Of the reasons maybe that the color force increases with distance and the energy barrier does not let us to pull a quark out of a particle e.x. (meson, baryon) by means of for example quantum mechanical tunneling, thus a free quark is not observed because by the time the pair-production energy for quark-antiquark pairs the increasing energy with distance is at rate about 1 GeV per fermi, however for (u) and (d) quarks the average distance in which the pair-production occurs is much less than a fermi. The force particles of QCD are gluons and they differ from the force particles of QED, the photons.

In one important respect, gluons can interact with each other, because of their color charge they are dynamical sources of color charges and quarks are statistical sources of color charges like photons all gluons have spin 1 and zero mass and in the QCD strong interactions between quarks. This gluons are intermediate particles, unlike photons (the self interactions) between gluons due to the nonabelian structure of  $SU(3)_c$  leads to a confined field lines between a quark and antiquark (in mesons) to string like flux tubes. For QCD interactions the field lines in an electric dipole fill the space between and around the charge.

The existence of these strings leads to a linearly increasing potential infinite energy being required to separate the quarks to infinity. In flux tube models the excitations of the flux tube can lead to exotic mesons, in the region from about 1.5 to 2.5 (proton masses), for which there is insufficient experimental evidences. These are gluonic excitations and there is theoretical and experimental efforts to understanding the spectrum and decay modes of them.

A deep understanding of gluon structure and properties like its propagator, .... needs to explain its behavior at high energy and low energy (quark-gluon) interactions. There is asymptotic freedom at high energy, the gluons are confined and asymptotic freedom of QCD interactions due to perturbative structure of nonabelian  $SU(3)_c$  theory. But what happens at low energy in which QCD is not perturbative and how gluons behaves at that region of energy (infrared slavery).

The non-perturbative features are closely related to bound state formed by strong forces such as mesons and baryons.

In fact this states are formed as the results of interactions which are present in final states of high energy scattering experiments. In this way, the gluon behaviors come to the focus of many theoretical attempts resulting in various phenomenological models. In bound state problems can be solved exactly and predictions can be made explicitly by calculating spin dependent forces since the potential form is well known. On the other hand, the potential form for bound states is not known in QCD and so there exist various potential models, moreover, the conventional perturbation theory in powers of inverse quark mass and strong coupling constant cannot be applied securely for low mass hadrons. Although the explicit form of confining potential cannot be obtained from a first principle formalism, the spin-dependent corrections to the static energy have been obtained by considering relativistic propagator expansions.

In the lagrangian field theories that combine the two phenomenological and field theoretical aspects of scaling phenomena

a) Parton model

b) Momentum dependent coupling constant by Feynman and Wilson respectively.

One must find a lagrangian which becomes asymptotically free. A theory is called asymptotically free if it has a fixed point as the momentum goes to infinity and if the coupling constant vanishes at this point. It has found that if Bjorken Scaling is accepted as the evidence for asymptotic freedom, no renormalizable field theory without non-Abelian gauge fields can be asymptotically free, thus there results the quark-gluon model for hadrons. In a conventional quark parton model there is an infinite number of quark separated in to valence and sea parts and the gluons had been generated but in a modified quark-parton model one assumes that the hadrons are composed of valence quarks and gluons only and the sea shows up via the pair production of gluons.

Now one can investigate the properties of gluon distributions such as momentum dependence and gluons have some contributions to experimental observations, one step to understanding gluon behavior at high and low energy regions and a way into resolving confinement problem. In investigations of the confinement mechanism, virtually all research terms, use a gauge theory instead of three, this makes the simulations more traceable, while preserving the essential properties of QCD. In these calculations the continuous space-time is replaced by a grid, or lattice, of points. The resulting theory is therefore

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known as a lattice gauge theory. In SU(2) there are three gauge fields carrying the color quantum number one can transform numerically as much of the physics as possible into one of the three SU(2) gauge fields.

This procedure has been used to test the dual superconductor ideas in SU(2) lattice gauge theory. This was inspired by the early ideas of Gerard 't Hooft, who suggested that magnetic monopoles were the key to understanding confinement and that only a subset of the original gauge fields would be needed. If monopoles are responsible for the existence of flux tubes they should also explain the masses of glueballs.

The so-called LSZ formalism the collision theory developed by Lehmann, Symanzik and Zimmermann together with perturbative theory describes the processes of elementary particles at high energies based on asymptotically free gauge theories.

The perturbative description of elementary particles is essentially based on the field-particle duality which means that each field in a quantum field theory is associated with a physical particle, this is not suitable for describing hadrons as composite states with quark gluonic substructure in hadron processes. We must extend scattering theory of composite particles described by (almost local) fields leading to a generalized LSZ formalism for bound states [1-3].

For a description of confinement of quarks and gluons within the framework of local quantum field theory, the elementary fields have to be divorced completely from a particle interpretation. What now is relating to the charge structure must be related to the empirical spectrum of particles. In other hand the perturbation theories which apply to Non-Abelian gauge theories suppose that there are not color-nonsinglet gauge covariant operators and hadrons are colorless particles to explain confinement mechanism, because of their perturbative structures are not suitable for describing the confinement mechanism, e.g. Kinoshita-Lee-Nauenberg theorem [6].

The dependence of the RG invariant confinement scale on the coupling and the renormalization scale near the ultraviolet fixed point is determined by [7]

$$\Lambda = \mu \exp \left( - \int^g \frac{dg'}{\beta(g')} \right) \xrightarrow{g \rightarrow 0} \mu \exp \left( - \frac{1}{2\beta_0 g^2} \right), \quad \beta_0 > 0$$

Then the ratios of all bound state masses are, at least in the chiral limit, determined independent of all parameters. Thus, to study the infrared behavior of QCD amplitudes non-perturbative methods are required and as singularities are anticipated a formulation in the continuum is desirable. In order to express vertex functions and higher n-point functions in terms of the elementary two-point functions, one can use the (DSEs) [8, 9] Dyson-Schwinger equations which are the equations of motion for QCD Green's functions and identities like as Slanov-Taylor.

There are solutions to DSEs of QCD for the gluon propagator in Landau gauge and axial vector gauge [14, 15]. The study of the DSEs for the quark propagator for the gluonic interactions of quarks [16] describe the chiral symmetry breaking and quark masses are generated dynamically. Confinement always

is described by absence of colored states from the asymptotic state space altogether.

This admits singularity structures of quark and gluon Green's functions that cannot occur in a local quantum field theory. By using of the continuum limit of the lattice formulation of quantum field theory and a proper definition of the generating functional beyond perturbation theory, the concepts such as gauge fixing, the presence of long-range correlations such as infrared divergences, the possibility of infrared slavery in QCD may be understood.

One of the manifestations of confinement from asymptotic freedom is the contradiction between the positivity of the gluon spectral density in Landau gauge with antiscreening. Considering the gluon propagator in the covariant gauge and in Euclidean momentum space one can construct the longitudinal parts of vertex functions (except at vanishing or infinite momenta) where they also contain the transverse components, there is distinction between confined and confining gluons which the first one is a gluon propagator which is infrared suppressed and do not propagate over long distance and an infrared suppressed propagator almost necessarily violates reflection positivity, but infrared enhanced gluonic correlations have been referred to as confining gluons. The area law and the singularity of gluon propagator in the infrared as  $1/k^4$  in some gauge is responsible for a linearly rising interquark potential model led to the (2-point) correlations at infrared region usually referred to (infrared slavery).

But QCD in the covariant gauge would be able to accommodate confined gluons in coexistence with an effective quark interaction which is confining, however the effective interaction thereby arises from combinations of different contributions. This holds independently of whether that combination in the end gives rise to an interaction as strong as the  $\sigma/k^4$  infrared-slavery model or not. Such effective, e.g. quark-current interactions should replace the literal meaning of a gluon propagator in the confinement argument. In this approach e.g. one can write the quark-two quark and etc. propagators which have no singularity and find DSEs solutions.

There are mainly two types of confinement potentials discussed in the literature, namely the two-body confinement potential introduced by Lipkin and the color flux tube (string) model of confinement. In the color flux tube model, the color string describes the hidden gluon degrees of freedom that are necessary to preserve color gauge invariance of the underlying field theory in a many quark system, or in the interaction region of hadron-hadron collisions, the strings can change their positions and oscillate between different configurations, this continuous flipping of color string can be effectively described in the flip-flop model which has been introduced to avoid the long-range color van der Waals forces in hadron-hadron interactions. In this model, the confinement interaction between any pair of quarks depends on the position of the remaining quarks, it thus contains many-body operators.

There is several two-body confinement potential models of Lipkin-type, which differ in their radial dependence but not in their color structure and are not adequate for describing the complicated dynamics of changing color strings in a many-

body (quark) system. QCD framework permits the classification of quarks and diquarks as constituent and current, while d-cons and q-cons play an important role in processes involving small values of the momentum transfer ( $q$ ), the role of these two types' entities in other ranges of  $q$ , can be depicted as follows

The processes involving

- i) intermediate values of  $q^2$  (d-cons  $\rightarrow$  d-curr+gluons+pairs)
- ii) Large values of  $q^2$  (q-cons  $\rightarrow$  q-curr+gluons+pairs)
- iii) Sufficiently large  $q^2$  (d-curr  $\rightarrow$  q-curr+gluons+pairs)
- iv)  $q^2 \rightarrow \infty$  (All d  $\rightarrow$  gluons)

Thus the properties of a diquark depends in addition of the properties of the quarks of which the diquark is formed on the values of the momentum transfer. In conventional physics diquark idea successfully explains several elementary particle processes, namely

- (i) Hadron production in  $e^+e^-$  collisions (ii) deep inelastic lepton-nucleon scattering (iii) hadron-hadron hard (large  $P$ ) as well as soft (low  $P$ ) collisions and other exclusive processes, (iv) supersymmetry of some mesons and baryons, (v) the properties of exotic baryons (pentaquark). Meanwhile the Lattice results do not support the concept of substantial diquark clustering (scalar) as an appropriate description of the internal Structure of low-lying hadrons. In diquark clustering it is well known that the one-gluon exchange hyperfine interaction has some relevance only when the quark masses are taken to be constituent quark masses, experimental mass splitting indicate it is not the relevant interaction between current quarks.

There is an infinite class of multiple gluon-exchange diquarks to consider beyond the exchange of a single gluon. Moreover these diagrams are equally important since the theory is nonperturbative. In a relativistic quark model [21] for the quark-quark (qq) interaction we have

$$V(P, q; M) = \bar{u}_1(p) \bar{u}_2(-p) v(P, q; M) u_1(p) u_2(-q)$$

With

$$v(P, q; M) = \frac{1}{2} \left[ \frac{4}{3} \alpha_s D_{\mu\nu}(k) \gamma_1^\mu \gamma_1^\nu + V_{conf}^V(k) \Gamma_1^\mu(k) \Gamma_{2\mu}(-k) + V_{conf}^S \right]$$

The Kernel  $v(P, q; M)$  is the quasipotential operator of the quark-quark or quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. For the quark-quark interaction in a diquark one can use the relation  $V_{qq} = V_{q\bar{q}}/2$  arising under the assumption about the octet structure of the interaction from the difference of the qq and  $q\bar{q}$  color states, for the confining interactions the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli terms.  $\alpha_s$  Is the QCD coupling constant  $D_{\mu\nu}$  is the gluon propagator in the coulomb gauge

$$D^{00}(k) = -\frac{4\pi}{k^2}, D^{ij}(k) = -\frac{4\pi}{k^2} \left( \delta^{ij} - \frac{k^i k^j}{k^2} \right), D^{0i} = D^{i0} = 0$$

And  $k=p-q$ ,  $\gamma_\mu$  and  $u(p)$  are the Dirac matrices and spinors

$$u^\lambda(p) = \sqrt{\frac{\varepsilon(p)+m}{2\varepsilon(p)}} \begin{pmatrix} \frac{1}{\sigma_p} \\ \frac{\sigma_p}{\varepsilon(p)+m} \end{pmatrix} X^\lambda$$

With

$$\varepsilon(p) = \sqrt{p^2 + m^2}.$$

The effective long-range vector vertex of the quark is defined by [21, 22].

$$\Gamma_\mu(k) = \gamma_\mu + \frac{ik}{2m} \sigma_{\mu\nu} \tilde{k}^\nu, \quad \tilde{k} = (0, k)$$

Where  $k$  is the Pauli interaction constant characterizing the anomalous chromo magnetic moment of quarks? In the configuration space the vector and scalar two-body confining potentials in the nonrelativistic limit reduce to

$$V_{conf}^V(r) = (1 - \varepsilon) V_{conf}(r),$$

$$V_{conf}^S(r) = \varepsilon V_{conf}(r),$$

With

$$V_{conf}(r) = V_{conf}^S + V_{conf}^V(r) = A_r + B,$$

Where  $\varepsilon$  is the mixing coefficient and A, B, are the parameters of the linear potential and have usual values of quark model. From the consideration of charmonium radiative decays  $\varepsilon = -1$  [23] and the universal Pauli interaction constant  $k = -1$  has been fixed from the analysis of the fine splitting of heavy quarkonia  $3P_J$ -states [23].

However one can consider the tHooft-like interaction between quarks induced by instantons and take the quark anomalous chromo magnetic moment  $k = -0.744$  (Diakonov) [24]. The one gluon exchange term in the nonrelativistic limit reduce to  $A/r$  where A depends on  $\alpha_s$  (strong coupling constant) and when uses for the interaction between a diquark and a quark or between two diquarks also depends on diquark form factor  $F(r)$ .

There is several examples of two-body confinement potentials [24].

$$V_{qq}^{conf}(\vec{r}_i, \vec{r}_j) = -a_c^q \lambda_i \cdot \lambda_j (\vec{r}_i - \vec{r}_j)^2$$

$$V_{qq}^{conf}(\vec{r}_i, \vec{r}_j) = -a_c^l \lambda_i \cdot \lambda_j |(\vec{r}_i - \vec{r}_j)|$$

$$V_{qq}^{conf}(\vec{r}_i, \vec{r}_j) = -a_c^r \lambda_i \cdot \lambda_j |(\vec{r}_i - \vec{r}_j)|^{2/3}$$

$\lambda_i$  And  $r_i$  are position and color matrices of quark.

This potential and a color-coulomb potential  $A/r$ , which describe the QCD strong interaction between quarks at high energy and A depends on  $\alpha_s$  (strong coupling constant), leads in the framework of the nonrelativistic Schrodinger equation to the observed linear Regge trajectories of hadron masses and finally we consider the color-screened error-function confinement

$$V_{qq}^{conf}(\vec{r}_i, \vec{r}_j) = -a_c^e \lambda_i \cdot \lambda_j \text{erf}(\mu r)$$

$r$  is the distance between two quarks

This potential rises linearly for small, but as a result of quark-antiquark pair production (color screening) grows only weakly for intermediate and finally goes to a constant value at large  $r$ . Lattice calculations show that such a behavior of the effective quark quark potential arises if quark-antiquark loops

are taken into account. The inverse of  $\mu$  is called color-screening length for which we take  $1/\mu \approx 0.8 \text{ fm}$ .

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